CCR - Competition Competence Report winter 2014/1

Merger Simulation Models: Part 4

This CCR deals with the use and application of AIDS and PCAIDS models. The simple version of the PCAIDS model is explained in the following by using a practical numerical example.

1. AIDS - Almost Ideal Demand System

The first step to any merger simulation is the specification of the demand function which depicts the interrelation between prices and quantity demanded. One of the most popular models for empirical demand analysis is the so-called **Almost Ideal Demand System** (AIDS) model. This demand system is based on the recognition that the "average" consumer behaves in a way so as to maximize utilities subject to budget constraints. The analysis is primarily based on the assumption of a particular class of preferences that can be aggregated to represent the demand decisions of this particular "average" consumer. These preferences lie on the cost function (expenditure function) which attributes certain expenditures at given relative prices to respective consumer utilities.

A distinct feature of the AIDS-model is that price and expenditure elasticities are not expected to be constant but instead vary with total expenditure. Accordingly, the richer consumers become the less luxury goods exist on the market. As opposed to other estimation methods, an increase in total expenditure does not lead to a rise but to a fall of the expenditure elasticities. This relationship is close to reality: the share of food expenditure is, for instance, not likely to rise with total expenditure. This property of the AIDS model is directly derived from the assumption of stable preferences.

A major advantage of AIDS-models lies in the fact that the demand functions derived from the utility functions are subject to numerous restrictions that can be empirically falsified. These restrictions are:

1. The adding-up-restriction, where the expenditure elasticities weighted against their respective budget shares sum to one;
2. The homogeneity restriction, where the expenditure-, price-, and cross-price elasticities sum to zero;
3. The symmetry requirement, where the compensated cross-price elasticities weighted against budget shares are identical;
4. The negativity restriction, where the compensated demand functions show a downward trend.

In addition, this system is indirect non-additive; this implies that consumption of one product can influence the marginal utility of another product.

From an econometric point of view, the AIDS-model holds the major advantage, relative to other demand systems, that the model can be almost ("*almost*") completely written in terms of linear equations. Based on expenditure shares and the estimated coefficients, price and expenditure elasticities can be easily calculated. Where a general price index depicting a linear homogenous function of the prices is used, estimating an AIDS demand function by using the OLS method is possible.

AIDS is a particular user-friendly system, which makes it well-suited for merger analyses without losing complexity compared to other estimation methods.

2. PCAIDS - Proportionality-Calibrated AIDS

A variant of the AIDS model is the Proportionality-Calibrated AIDS (PCAIDS)-model. These models are especially valuable when there are data limitations or estimation problems, or when a rapid and less costly analysis is required.

It requires information only on

(1) market shares,
(2) the industry price elasticity[1], and
(3) the price elasticity for one brand in the market.[2]

The logic of PCAIDS is simple. The share lost as a result of a price increase is allocated to the other firms in the relevant market in proportion to their respective shares.

3. Practical merger example

The following example illustrates a hypothetical merger between firms producing socks. The market consists of three active producers, on producing blue ballerina socks, the other one green knee-high socks and the third one producing red sport socks.

The manufacturers of blue ballerina and green knee-high socks merge. How does a merger between those two companies affect prices post-merger?

In this example, we make the following assumptions. They can easily be replaced with real data.

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1 The industry price elasticity shows the percentage change in sales in response to a one percent increase in price when all firms belonging to that industry simultaneously increase their price.

2 The own-price elasticity of a particular good gives the percentage change in the good´s sales volume in response to a one percent increase in its price.

- **Model assumption 1**

  The market shares of the three brands are distributed as follows: 20% for the blue ballerina socks, 30% for the green knee-high socks and 50% for the red sport socks.

- **Model assumption 2**
  We assume that competition in the socks market is best described by the Bertrand model with differentiated products. This model states that when a firm raises its price above that of its competitors, it would still retain customers who are willing to buy its product because they value the product features more than the price increase.
- **Model assumption 3**
  The demand function is specified by the PCAIDS model. This model only requires information about each firm's market share, the industry price elasticity and the own-price elasticity of one firm.

These assumptions are incorporated in the model under the form of mathematical equations.

3.1. Pre-merger

Equations 1, 2 and 3 displayed in figure 1 indicate how each product's change in market share is affected by changes in the product's price as well as competing products' prices. In the equations, **d** stands for "*change in*", **S** refers to "*market share*" and **dS** is the "*change in market share*". **dp/p** refers to the percentage change in the price, where **p** stands for "*price*".

Figure 1: Simulation of market share changes for blue, green and red socks

Equation 1:

$$dS_{blue} = -0{,}4\left(\frac{dP_{blue}}{P_{blue}}\right) + 0{,}15\left(\frac{dP_{green}}{P_{green}}\right) + 0{,}25\left(\frac{dP_{red}}{P_{red}}\right)$$

Equation 2:

$$dS_{green} = 0{,}15\left(\frac{dP_{blue}}{P_{blue}}\right) - 0{,}525\left(\frac{dP_{green}}{P_{green}}\right) + 0{,}375\left(\frac{dP_{red}}{P_{red}}\right)$$

Equation 3:

$$dS_{red} = 0{,}25\left(\frac{dP_{blue}}{P_{blue}}\right) + 0{,}375\left(\frac{dP_{green}}{P_{green}}\right) - 0{,}625\left(\frac{dP_{red}}{P_{red}}\right)$$

Reading example: The first equation displays the demand function for blue socks. When the price of blue socks increases by 1% (dp_{blue}/p_{blue} = 0,01), the market share of the blue socks (dS_{blue}) will change by -0,4% i.e. the market share of the blue socks manufacturer decreases by 0,4% to the benefit of the two competing products.

The lost sales of blue socks are shifted to the green and red socks in proportion to market shares. More specifically, because the red brand's market share is 1,667 times larger than the green brand's market share (50/30=1,667), the change in market share of the red brand will also be 1,667 times bigger than the change in market share of the green brand (0,25/0,15=1,667). Thus, the market shares of green and red socks increase by 0,15% and 0,25% respectively.

This model therefore entails the **proportionality assumption**: whenever the price of one brand increases, consumers will switch to the two competing brands in proportion to market shares.

The following table depicts the own- and cross-price coefficients extracted from equations 1-3.

The own-price elasticity measures how a one percent price increase of one product affects the quantity demanded for that same product. The cross-price elasticity, in turn, measures how a one percent price increase of one product affects the quantity demanded for another product. Each brand's own-price coefficients are marked in their respective color, while the cross-price coefficients are highlighted in yellow.

Table 1: Overview of own- and cross-price coefficients

Socks	Price coefficients		
	blue	**green**	**red**
blue	**-0,4**	**0,15**	**0,25**
green	**0,15**	**-0,525**	**0,375**
red	**0,25**	**0,375**	**-0,625**

- **Model assumption 4**

  The value of the industry price elasticity (ε) is -1. This means that if the prices of all three brands were to increase simultaneously by one percent, the demand for the three brands would decrease each by one percent.

Figure 2 displays the calibration formulas for elasticities used in the PCAIDS model. In these equations b_i refers to the own-price coefficient of brand i, b_{ij} refers to the cross-price coefficient of brand i with respect to brand j, S_i is brand i's market share and ε is the industry price elasticity.

Figure 2: Pre-defined formulas for own- and cross-price elasticities in the PCAIDS model

Equation 4 - Own-price elasticity: $\varepsilon_i = -1 + \frac{b_i}{S_i} + S_i(\varepsilon + 1)$

Equation 5 - Cross-price elasticity: $\varepsilon_{ij} = \frac{b_{ij}}{S_i} + S_i(\varepsilon + 1)$

In equation 6 (figure 3), the following inputs are used in equation 4 to calculate the own-price elasticity of green socks:

- the own-price coefficient shown in table 1: -0,525
- the market share of the green brand: 30%
- as well as the industry price elasticity: ε = -1

Figure 3: Own-price elasticity of green socks

Equation 6: $\varepsilon_{green} = -1 + \frac{-0{,}525}{0{,}3} + 0{,}3 \times (-1 + 1) = -2{,}75$

This value indicates that as the price of green socks increases by 1%, the demand for green socks will drop by 2,75%.

In equation 7 (figure 4), we insert the appropriate values in equation 5 to calculate the cross-price elasticity of the green socks with respect to the price of the blue socks.

Figure 4: Cross-price elasticity of green socks with respect to blue socks

Equation 7: $\varepsilon_{greenblue} = \frac{0{,}15}{0{,}3} + 0{,}3 \times (-1 + 1) = 0{,}5$

This value indicates that as the price of blue socks increases by 1%, the demand for green socks will increase by 0,5%.

After having run these calculations for all possible product combinations, we obtain the following own- and cross-price elasticities, summarized in Table 2. Own-price elasticities are highlighted in their respective colors, cross-price elasticities are highlighted in yellow.

Table 2: Own- and cross-price elasticities <u>before the merger</u>

Socks	Elasticities with respect to		
	p_{blue}	P_{green}	P_{red}
blue	**-3**	**0,75**	**1,25**
green	**0,5**	**-2,75**	**1,25**
red	**0,5**	**0,75**	**-2,25**

3.2. Post-merger

After the merger, the market shares change in the following way: the blue brand has a market share of 25%, whereas the green and the red brands have market shares of 35% and 40% respectively. Calculations with these new market shares yield the following post-merger price elasticities.

Table 3: Own- and cross-price elasticities <u>after the merger</u>

Socks	Elasticities with respect to		
	p_{blue}	P_{green}	P_{red}
blue	**-2,6**	**0,6**	**1**
green	**0,43**	**-2,5**	**1,07**
red	**0,625**	**0,94**	**-2,56**

- **Model assumption 5**
  Each firm acts as a profit maximizer. In the PCAIDS model, this means that the Lerner Index formula is satisfied. The Lerner Index depicts the relationship between a firm's own price elasticity and its mark-up.

In equation 8, the own-price elasticity of firm I, written as ε_i, is used to calculate the pre- and post-merger prices of firm I, written as p_i, under the assumptions of profit maximization and constant marginal cost. More specifically, we assume that marginal cost of firm i, written as MC_i, is equal to 1 for each firm and it does not change after the merger.

$$\textbf{Equation 8: } \frac{-1}{\varepsilon_i} = \frac{p_i - MC_i}{p_i}$$

We can now extract p from the equation. In order to calculate the price difference, we insert the respective price elasticities of blue and green socks into equation 8, which yields the profit margins pre- and post-merger. Comparing these two profit margins lets us determine the percentage change in prices pre- and post-merger.

$$\frac{post\ merger\ prices - pre\ merger\ prices}{pre\ merger\ prices} = unilateral\ effects$$

Our PCAIDS simulation predicts a unilateral post-merger price increase (absent efficiencies) of **8,33% for blue socks** and **6,06% for green socks** absent any cost efficiencies.